Math 72 6.3 Adding and Subtracting Rational Expressions

Objectives

1) Find common denominators
2) Rewrite equivalent fractions with common denominators
   → "higher terms", opposite of "simplify."
3) Add or subtract rational expressions
4) Use the order of operations for rational expressions containing add, subtract, multiply or divide.

Perform the indicated operations, then fully simplify if possible.

① $\dfrac{2x^2+x}{x^2-4} + \dfrac{x-x^2}{x^2-4}$

- add two fractions, must have common denominator.
- both fractions have denominator x^2-4 ☺
- add numerators
- use common denominator, unchanged.

$= \dfrac{2x^2+x + x-x^2}{x^2-4}$ ← add numerators
← common denominator

$= \dfrac{x^2+2x}{x^2-4}$ ← combine like terms

$= \dfrac{x\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$ ← factor GCF x
← factor difference of squares

$= \boxed{\dfrac{x}{x-2}}$ cancel $\dfrac{x+2}{x+2} = 1$

Easier example:

$\dfrac{2}{15} + \dfrac{8}{15}$

$= \dfrac{2+8}{15}$ ← add numerators
← common denominator

$= \dfrac{10}{15}$ ← combine like terms

$= \boxed{\dfrac{2}{3}}$ ← factor & cancel = reduce

(2) $\dfrac{b^2-11}{b^2-25} - \dfrac{3b+1}{25-b^2}$

- subtract two fractions, must have common denominator
- $b^2-25 \neq 25-b^2$. We do not have a common denom.
- factor -1 from $25-b^2$

$$= -1(-25+b^2)$$
$$= -(-25+b^2)$$
$$= -(b^2-25)$$

$$= \frac{b^2-11}{b^2-25} - \frac{3b+1}{-(b^2-25)}$$

↑ move this negative out of the denominator

simpler example $\dfrac{1}{-5} = -\dfrac{1}{5} = \dfrac{-1}{5}$

↑ denom ↑ front ↑ numerator

$(-)\cdot(-)=(+)$

$$= \frac{b^2-11}{b^2-25} - \frac{-(3b+1)}{b^2-25}$$ ← common denom! ☺

$$= \frac{b^2-11}{b^2-25} + \frac{3b+1}{b^2-25}$$

$$= \frac{b^2-11+3b+1}{b^2-25}$$ ← add numerators ← keep common denom

$$= \frac{b^2+3b-10}{b^2-25}$$ ← combine like terms

-10, 5, -2, 3

$b^2+3b-10$
$= (b+5)(b-2)$

$$= \frac{(\cancel{b+5})(b-2)}{(b-5)(\cancel{b+5})}$$ ← factor difference of squares

$$= \boxed{\frac{b-2}{b-5}}$$ simplify by cancel $\dfrac{b+5}{b+5} = 1$

(3) $\dfrac{4x-1}{4x^2+8x+4} - \dfrac{2x-3}{2x^2-2x-4}$

- subtract, need common denom
- factor each denom completely

$$4x^2+8x+4 = 4(x^2+2x+1) = 4(x+1)(x+1) \quad \text{GCF } 4$$

$$2x^2-2x-4 = 2(x^2-x-2) = 2(x-2)(x+1) \quad \text{GCF } 2$$

$$= \frac{4x-1}{4(x+1)(x+1)} - \frac{2x-3}{2(x-2)(x+1)}$$

need <u>LOWEST</u> common denominator.

Note: $4\cdot 2\cdot (x+1)(x+1)\cdot(x-2)(x+1)$ is a common denom, but it's <u>huge</u>! It's not the lowest.

The first denom is missing $(x-2)$ compared to second denom
The second denom is missing $2(x+1)$ compared to first denom.
$= 2x+2$

$$= \frac{(4x-1)}{4(x+1)(x+1)}\cdot\frac{(x-2)}{(x-2)} - \frac{(2x-3)}{2(x-2)(x+1)}\cdot\frac{2x+2}{2(x+1)}$$

multiply by 1 using missing factors

multiply by 1 using missing factors

$$= \frac{(4x-1)(x-2)}{4(x+1)^2(x-2)} - \frac{(2x-3)(2x+2)}{4(x+1)^2(x-2)} \quad \leftarrow \text{common denom!}$$

$$= \frac{(4x-1)(x-2)-(2x-3)(2x+2)}{4(x+1)^2(x-2)} \quad \leftarrow \text{subtract numerators}$$

$$= \frac{(4x^2-8x-x+2)-(4x^2+4x-6x-6)}{4(x+1)^2(x-2)} \quad \leftarrow \text{simplify numerators - order of op, FOIL = mult}$$

$$= \frac{4x^2-9x+2-(4x^2-2x-6)}{4(x+1)^2(x-2)} \quad \leftarrow \text{combine like terms}$$

* must have ( ) to dist neg next

$$= \frac{4x^2 - 9x + 2 - 4x^2 + 2x + 6}{4(x+1)^2(x-2)}$$

$$= \boxed{\frac{-7x+8}{4(x+1)^2(x-2)}}$$

$$= \boxed{\frac{-(7x-8)}{4(x+1)^2(x-2)}}$$

factor out neg = best answer

④ $\dfrac{b}{b^2-25} - \dfrac{5}{b+5} + \dfrac{6}{b}$

- add and subtract fractions $\Rightarrow$ must have common denom
- add and subtract $L \to R$ by order of operations.

$$= \frac{b}{(b+5)(b-5)} - \frac{5}{(b+5)} + \frac{6}{b} \quad \leftarrow \text{factor all denoms}$$

denom missing b

denom missing $b(b-5)$

denom missing $(b+5)(b-5) = (b^2-25)$

$$= \frac{b}{(b+5)(b-5)} \cdot \frac{b}{b} - \frac{5}{(b+5)} \cdot \frac{b(b-5)}{b(b-5)} + \frac{6}{b} \cdot \frac{b^2-25}{(b+5)(b-5)}$$

mult by 1   mult by 1   mult by 1 $\leftarrow$ use missing factors

$$= \frac{b \cdot b}{b(b+5)(b-5)} - \frac{5b(b-5)}{b(b+5)(b-5)} + \frac{6(b^2-25)}{b(b+5)(b-5)}$$

$$= \frac{b^2 - 5b(b-5) + 6(b^2-25)}{b(b+5)(b-5)} \quad \leftarrow \text{add \& subtract numerators}$$

$$= \frac{b^2 - 5b^2 + 5b + 6b^2 - 150}{b(b+5)(b-5)} \quad \leftarrow \text{order of op: multiply before add or subtract}$$

$$= \frac{2b^2 + 5b - 150}{b(b+5)(b-5)} \quad \leftarrow \text{combine like terms}$$

Factor numerator $2b^2 + 5b - 150$

-300

-15   20

5

-1, 300
-2, 150
-3, 100
-4, 75
-5, 60
-6, 50
-10, 30
-12, 25
-15, 20 ✓

$$2b^2 - 15b + 20b - 150$$
$$= b(2b-15) + 10(2b-15)$$
$$= (2b-15)(b+10)$$

$$= \boxed{\frac{(2b-15)(b+10)}{b(b+5)(b-5)}}$$

nothing cancels!

* LEAVE FINAL ANSWER FACTORED WHEN IT'S A FRACTION *

(5) $$\frac{a+3}{a-3} - \frac{a+3}{a-3} \cdot \frac{a^2-4a+3}{a^2+5a+6}$$

↑ subtract ↑ multiply

* ORDER OF OPERATIONS *
Multiply BEFORE SUBTRACT.

$$= \frac{a+3}{a-3} - \underbrace{\frac{\cancel{(a+3)}}{\cancel{(a-3)}} \frac{\cancel{(a-3)}(a-1)}{\cancel{(a+3)}(a+2)}}_{\text{multiply is factor and cancel}}$$

3, −3, −1, −4

6, 3, 2, 5

$$= \frac{a+3}{a-3} - \frac{a-1}{a+2}$$

↑ subtract, need lowest common denominator $(a-3)(a+2)$

$$= \frac{a+3}{a-3} \cdot \frac{a+2}{a+2} - \frac{a-1}{a+2} \cdot \frac{a-3}{a-3}$$

↑ multiply by 1 ↑ multiply by 1

$$= \frac{(a+3)(a+2) - (a-1)(a-3)}{\quad}$$ ← subtract numerators

$$= \frac{a^2+5a+6 - (a^2-4a+3)}{(a+2)(a-3)}$$ ← simplify numerators
order of op – mult (FOIL)
* must have ( ) to dist neg *

$$= \frac{a^2+5a+6 - a^2+4a-3}{(a+2)(a-3)}$$ ← dist neg

$$= \frac{9a+3}{(a+2)(a-3)}$$ ← Combine like terms

$$= \boxed{\frac{3(3a+1)}{(a+2)(a-3)}}$$

leave final answer factored

⑥ $\left(\dfrac{x}{x+3} - \dfrac{x}{x-3}\right) \div \dfrac{x}{7x+21}$

↑ ↑

PARENTHESES first in ORDER OF OPERATIONS

subtract requires lowest common denominator $(x+3)(x-3)$

$$= \left(\frac{x}{x+3} \cdot \overset{\text{mult by 1}}{\frac{x-3}{x-3}} - \frac{x}{x-3} \cdot \overset{\text{mult by 1}}{\frac{x+3}{x+3}}\right) \div \frac{x}{7x+21}$$

$$= \frac{x(x-3) - x(x+3)}{(x-3)(x+3)} \div \frac{x}{7x+21}$$

simplify numerator
dist x and $-x$

$$= \frac{x^2 - 3x - x^2 - 3x}{(x-3)(x+3)} \div \frac{x}{7x+21}$$

$$= \frac{-6x}{(x-3)(x+3)} \cdot \frac{7x+21}{x}$$

combine like terms
mult by reciprocal

$$= \frac{-6}{(x-3)\cancel{(x+3)}} \cdot \frac{7\cancel{(x+3)}}{x}$$

multiply is factor and cancel
factor GCF 7

$$= \boxed{\frac{-42}{x(x-3)}}$$

cancel $\dfrac{(x+3)}{(x+3)} = 1$

mult $(-6)(7)$.

Math 70

(8) Simplify.

$$\frac{-y^3z+2y^2z^2-15yz^3}{z^2y-2y^2z} + \frac{y^3z-2y^2z^2-8yz^3}{2y^3z+3y^2z^2-2yz^3} \div \frac{16z^2-y^2}{y^3+64z^3}$$

↑ add ↑ divide

Order of operations: divide before add. Mult by reciprocal:

$$= \frac{-y^3z+2y^2z^2-15yz^3}{z^2y-2y^2z} + \underbrace{\frac{y^3z-2y^2z^2-8yz^3}{2y^3z+3y^2z^2-2yz^3} \cdot \frac{y^3+64z^3}{16z^2-y^2}}$$

To multiply, factor each completely.

Factoring: $y^3z-2y^2z^2-8yz^3$

GCF $= yz(y^2-2yz-8z^2)$

trinomial leading coef 1

-8 ← factors mult to -8

-4 2

-2 ← factors add to -2

$$= \underline{\underline{yz(y-4z)(y+2z)}}$$

$2y^3z+3y^2z^2-2yz^3$

GCF $= yz(2y^2+3yz-2z^2)$

trinomial leading coef ≠ 1 "double X" = reminder to rewrite & group

$2(-2)$ ← a·c #s mult to -4

-4

4 -1

3 ← b #s add to 3

$$= yz\{2y^2 + \underbrace{4yz - yz}_{} - 2z^2\}$$

was $3yz$, now rewritten using grouping

$= yz\{2y(y+2z) - z(y+2z)\}$

$= yz\{(y+2z)(2y-z)\}$

$= \underline{\underline{yz(y+2z)(2y-z)}}$

Math 70

(B) con't

Factoring con't.

$y^3 + 64z^3$ sum of cubes

$\uparrow \quad \uparrow$

$(y)^3 \quad (4z)^3$

$a^3 + b^3 = (a \underset{\text{S}}{+} b)(a^2 \underset{\text{O}}{-} ab \underset{\text{AP}}{+} b^2)$

$= \underline{\underline{(y+4z)(y^2 - 4yz + 16z^2)}}$

$16z^2 - y^2$ difference of squares

$= \underline{\underline{(4z-y)(4z+y)}}$

Rewrite w/ factoring:

$$= \frac{-y^3z + 2y^2z^2 - 15yz^3}{z^2y - 2y^2z} + \frac{\cancel{yz}(y-4z)\cancel{(y+2z)}}{\cancel{yz}\cancel{(y+2z)}(2y-z)} \cdot \frac{\cancel{(y+4z)}(y^2 - 4yz + 16z^2)}{(4z-y)\cancel{(4z+y)}}$$

Divide out common factors: $y+4z = 4z+y$ can be cancelled

$4z - y = -y + 4z = -(y-4z)$

must factor out (-1) to cancel!

$$= \frac{-y^3z + 2y^2z^2 - 15yz^3}{z^2y - 2y^2z} - \frac{(\cancel{4z-y})}{(2y-z)} \cdot \frac{(y^2 - 4yz + 16z^2)}{(\cancel{4z-y})}$$

$$= \frac{-y^3z + 2y^2z^2 - 15yz^3}{z^2y - 2y^2z} - \frac{y^2 - 4yz + 16z^2}{2y-z}$$

To subtract, need a common denominator

Factor this denominator

$z^2y - 2y^2z$

GCF $= yz(z - 2y)$

$2y - z \neq z - 2y$! Must factor out -1

(or mult: $\frac{-1}{-1} = +1$)

$$= \frac{-(-y^3z + 2y^2z^2 - 15yz^3)}{-yz(z-2y)} - \frac{y^2 - 4yz + 16z^2}{2y-z}$$

Math 70

(B) cont

$$= \frac{y^3z - 2y^2z^2 + 15yz^3}{yz(2y-z)} - \frac{y^2 - 4yz + 16z^2}{(2y-z)}$$

* Astute observers might see a lot of yz in the first denominator and another method....

$$LCD = yz(2y-z)$$

$$= \frac{y^3z - 2y^2z^2 + 15yz^3}{yz(2y-z)} - \frac{yz(y^2 - 4yz + 16z^2)}{yz(2y-z)}$$

↑ dist $-yz$ to all terms in numerator

$$= \frac{\cancel{y^3z} - 2y^2z^2 + 15yz^3 - \cancel{y^3z} + 4y^2z^2 - 16yz^3}{yz(2y-z)}$$

combine like terms

$$= \frac{2y^2z^2 - yz^3}{yz(2y-z)}$$

← factor numerator

$$= \frac{yz^2(2y-z)}{yz(2y-z)}$$

divide out common factors

$$= \boxed{z}$$